Collaborative Research in Mathematics at Agnes Scott

Interested in working with professors at Agnes Scott College? Collaborative research in mathematics is simply that: collaborative. You will not be assisting in a lab; you will be leading with your mind. Some possible topics appear below.

Most students work with the professors during the academic year. Work over the summer may be possible, but at this point funding does not seem likely.

Recursive sequences modulo a prime

Take a sequence of integers with the property that each term can be expressed using the two previous. For example,

1, 4, 5, 9, 14, 23, 37, 60, …

In this sequence, each term is the sum of the two previous terms. Next, consider the remainder of each term when it is divided by a prime number. For example, if we divide by 11 the remainder of each term in the above sequence is

1, 4, 5, 9, 3, 1, 4, 5, …

This is called the sequence modulo 11. What you see is that this new sequence becomes periodic, and its period is 5 since it repeats after every five terms. Undergraduate researchers such as you have investigated questions about this periodic behavior, such as:

- Is the length of the period predictable?
- For a given number n, can we construct a sequence which, modulo p for some prime, has period length n?

Problems in this area, loosely defined, have been investigated by Katy Brooke ‘18, Aubrey Coffey ‘17, Denisse Saucedo ‘17, Biru Tang ‘17, Cassie Xu ‘16, Meina Zhou ‘14, Shan Shan ‘14, Chrissy Franzel ’13, Hilary Tobiasz ’12, Rose Psalmond ’11, and Chuya Guo ‘09. Brooke, Saucedo, and Xu have written a paper, and Coffey and Tang have written another, and both will appear in the Proceedings of the 14th Annual Harriet J. Walton Symposium for Undergraduate Research Chuya Guo and Alan Koch coauthored the paper *Bounds for Fibonacci period growth*, involve, **2** no. 2 (2009), 195-210.

Students should have completed MAT 206 (Linear Algebra) and, preferably, MAT 204 (The Art of Mathematical Thinking).

Contact Alan Koch (akoch@agnesscott.edu) if you are interested.

Chaotic Dynamical Systems

Dr. Wiseman works in chaotic dynamical systems. His current project involves dividing a system into overlapping regions and studying the interaction between the regions. One can then use linear

algebra to extract information about the system (for example, the existence of chaotic behavior). If this sounds interesting, and you've taken linear algebra (math 206), you should talk to him (jwiseman@agnesscott.edu).

Mathematical Biology

Occasionally, Alan Koch (Mathematics) and Lock Rogers (Biology) develop projects in modeling evolutionary biology. Currently, Emily Bryans '18 is using Game Theory to study the tradeoff between resource allocation for reproduction and resource allocation for survival. Jordan Kennedy '12 studied protogynous hermaphrodites, species which are born one gender and switch at some point in their lifetimes, where questions concerning the optimal time to change gender (with the goal of maximizing reproduction).

Students interested in these questions should have completed or be currently taking MAT 119 (Calculus II) and have a working understanding of probability.

This research is the combined effort of Alan Koch (akoch@agnesscott.edu) and Lock Rogers (lrogers@agnesscott.edu). Contact either if you are interested.

p-adic Geometry

Typically, given a real number r we define its absolute value, $|r|$ to be the usual distance of r to the origin on a number line. Absolute value then gives rise to the distance between two numbers: the distance between x and y on the number line is $|x - y|$. The notion of distance is closely connected to the geometry we know and love: many geometric objects have definitions that depend on distance, for example a circle with center c and radius r is the set of points in the plane which are distance r units from c.

Now, for a fixed prime p, define the p-adic absolute value of an integer n to be $|n|_p = p^{-v}$, where v is the nonnegative integer with the property that p^v is a factor of n and p^{v+1} is not. For example, $|54|_3 = 1/27$ since 27 is a factor of 54 but 81 is not. Notice also that $|54|_2 = ½$ and $|54|_5 = 1$.

With this new absolute value, we get a new notion of distance, allowing us to obtain a new geometry. In this geometry, all triangles are isosceles, there are no rectangles, and circles have more than one center. What are some of the other properties of this geometry?

Emily Piff '17 is current investigating this; previous researchers include Elle O'Brien '13 and Catherine Crompton '06. Crompton's paper, *Some Geometry of the* p*-adic Rationals*, can be found in the Rose-Hulman Undergraduate Mathematics Journal , vol. 8 2007.

Students interested in this area should have completed or be currently taking MAT 204 (The Art of Mathematical Thinking) and the ability/desire to think about some really weird things.

Interested students should contact Alan Koch (akoch@agnesscott.edu).